Stochastic forcing of north tropical Atlantic sea surface temperatures by the North Atlantic Oscillation

Cécile Penland[1] and Leslie M. Hartten[2, 1]

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Corresponding author: C. Penland, NOAA/ESRL/Physical Sciences Division, R/PSD3, 325 Broadway, Boulder CO 80305, USA (Cecile.Penland@noaa.gov)

[1] NOAA/Earth System Research Laboratory/Physical Sciences Division, Boulder, Colorado, USA
[2] Cooperative Institute for Research in Environmental Sciences (CIRES), University of Colorado, Boulder, Colorado, USA

Key Points:

- Stochastic forcing affects tropical Atlantic SST's seasonal predictability
- Much of this stochastic forcing is due to the North Atlantic Oscillation (NAO)
- The NAO's effect on evolution of north tropical Atlantic SST is quantified

Abstract

The North Atlantic Oscillation (NAO) is a rapidly decorrelating process that strongly affects the climate over the Atlantic and the surrounding continents. Although the NAO itself is basically unpredictable on seasonal timescales using statistical methods, NAO forcing is here shown to significantly affect sea surface temperatures (SSTs) evolving on those timescales. Results using Linear Inverse Modeling (LIM) imply that the NAO index and its convolution with deterministic SST dynamics account for nearly half the unpredictable component of north tropical Atlantic SST at lead times greater than nine months; adding this component to hindcasts at a lead of 48 weeks increases correlation with north tropical Atlantic SST from about 0.4 to about 0.6. Rapid fluctuations during boreal winter and spring, when the NAO is strongest, affect SST predictability throughout the entire year.

Index Terms

Ocean/atmosphere interactions, Stochastic processes, Seasonal Predictability

1. Introduction

Seasonal climate predictability in the Atlantic-rim nations is strongly influenced by tropical Atlantic variability [*Hastenrath,* 1984; *Enfield,* 1996; *Nobre and Shukla,* 1996; *Polo et al.,* 2008; *Kushnir et al.,* 2010]. Thus, improved forecasts of sea surface temperature (SST) in this region [*Stockdale et al.*, 2006; *Kushnir et al.,* 2006], particularly in the north tropical Atlantic (NTA) where hurricanes form [*Goldenberg and Shapiro,* 1996; *Elsner et al.,* 2013], may improve predictability over land on seasonal scales.

Air-sea interactions influencing monthly NTA SSTs have been shown to involve several remote, possibly inter-related phenomena such as El Niño [*Enfield and Mayer*, 1997; *Saravanan and Chang*, 2000; *Czaja et al.*, 2002], the Pacific-North American (PNA) pattern and the North Atlantic Oscillation (NAO). Monthly variations in NTA SSTs are influenced in part by the effect on Atlantic trade winds of Rossby-wave propagation associated with these teleconnections [*Lee et al.* 2008]. This study is concerned with a subset of these influences, i.e., sub-monthly forcing of NTA SSTs by the NAO.

The NAO has a short decorrelation time [*Wu and Liu* ,2002; *Kushnir et al.*, 2006; *Smirnov and Vimont*, 2013], much shorter than that of SST anomalies in NTA (Fig. 1a). Synoptic-scale wave breaking associated with both the origin and decay of the NAO [*Feldstein*, 2007; *Benedict et al.*, 2004; *Franzke et al.*, 2004] is a strongly nonlinear mechanism operating on climatically short timescales of about 10 days [*Feldstein*, 2000]. However, when a dynamical system comprises coupled processes evolving on separate timescales, the slow process (like SSTs) can "see" a broadband, rapidly decorrelating process (like the NAO) as stochastic forcing.

The idea that rapidly varying atmospheric variability may act as stochastic forcing of slow oceanic processes has been extensively discussed in the climate literature, beginning with articles by *Hasselmann* [1976] and *Frankignoul and Hasselmann* [1977]. The theoretical justification for this idea is the dynamical Central Limit Theorem [*Khasminskii*, 1966; *Papanicolaou and Kohler*, 1974; *Gardiner*, 1984], which requires that a system be forced by a combination of strong but weakly correlated influences operating on much faster timescales than the forced system. Stochastic forcing of Atlantic SST has been discussed previously in a landmark paper by *Frankignoul et al.* [1998], who used a one-dimensional mixed layer model to investigate the stochastic forcing of SST by local air-sea fluxes in a region of the Atlantic considerably northward of our interest. However, a one-dimensional model is not justified in the NTA region, where mean surface currents can be considerable.

The effects of stochastic atmospheric forcing on SST are manifold: 1) Whether or not self-sustaining oceanic variability exists, stochastic forcing modifies the internal dynamical behavior of the oceanic system. Furthermore, when self-sustaining oceanic variability does not obtain, stochastic forcing maintains the system. 2) The large spatial scale of stochastic forcing related to the NAO can generate correlations between SSTs at locations that are not directly connected by currents. 3) An unpredictable component of SST is generated. (Here, we mean "unpredictable" in a pathwise sense; probabilistic forecasts of this component are not precluded.) This article investigates this last effect, concentrating on SSTs in the north tropical Atlantic (SST_{NTA}) on seasonal timescales and employing statistical products at two different temporal resolutions.

We identify the fraction of that unpredictable component related to the rapidly varying NAO in the context of the best-fit linear model of tropical SSTs. Such Linear Inverse Models (LIMs: c.f. *Penland and Sardeshmukh* [1995]) are competitive with state of the art GCMs [*Saha et al.*, 2006]. First we use LIM to derive the linear dynamical description of SST anomalies

(SSTA) in the global tropical strip from seasonally averaged analyses. Although this linear description is based solely on SST, it implicitly includes other processes that may be parameterized by SST. Thus, the deterministic part of this model includes dynamics occurring as a result of the persistent El Niño phenomenon [*Penland and Matrosova,* 1998]. Next, we use those results to estimate time series of the corresponding stochastic forcing from weekly SSTA. We show that the SSTA in the NTA region ($SSTA_{NTA}$) include a substantial unpredictable component due to the NAO at lead times approaching a year. We further show that the NAO cannot be used in isolation to estimate this component, but must be convolved with the SST dynamics.

2. Determining the linear dynamics stochastic forcing in the tropical strip

We want to investigate how stochastic forcing affects the predictability of seasonal SST. How is the stochastic forcing time series estimated? Let **X** represent a low-resolution time series of SSTAs, such as three-month averages, and **x** a more finely resolved SSTA time series, such as weekly averages. Here "anomalies" are created by subtracting the average of each point in the annual cycle from the full series (the "anomaly method" [*Fuenzalida and Rosenblüth*, 1986], e.g. subtracting the DJF-average SST from each DJF SST); both **X** and **x** are therefore centered about 0. We use weekly averaged values for the finely resolved data because their SST values are as close as possible to "instantaneous" while providing good geographical coverage and reasonable instrumentation error. These weekly data allow us to sample phenomena unresolved on seasonal timescales and to verify that the timescales of the rapid dynamics are appropriately short, similar to NAO timescales.

Separating linear and nonlinear contributions to the tendency equation for the i^{th} component of **x**, we write

$$\frac{dx_i}{dt} = \sum_j L_{ij} x_j + N_i(\mathbf{x},t), \quad (1)$$

where **L** is a linear operator representing slow dynamics, defined here as monthly to seasonal timescales and longer, and **N** represents rapidly varying dynamics. *Penland and Matrosova* [1998] showed that the effect of El Niño on NTA forecasts is contained in **L**. If the decorrelation timescales of **N**(**x**,*t*) are shorter than three months, then applying a three-month mean to equation (1) combines decorrelated values of **N**(**x**,*t*), which, by the Central Limit Theorem, results in a term that is approximately Gaussian white noise, denoted $\boldsymbol{\xi}(t)$:

$$\frac{dX_i}{dt} = \sum_j L_{ij} X_j + \xi_i(t) \quad . \quad (2)$$

For our slowly varying SSTs, we use monthly Extended Reconstructed SST (ERSST [*Smith et al*., 2008]) data from 1950-2000. We estimate the constant **L** from the statistics of **X** using equation (2) as an *ansatz*; see the supplementary material for details. Our task is now to exploit the equivalence of $\boldsymbol{\xi}(t)$ in equation (2) and **N**(*t*) in equation (1), and investigate the NAO contribution to this forcing.

Given **L** (with units adjusted appropriately) and dropping explicit dependence of **N** on **x**, the quantity **N(***t***)** can be estimated from weekly SSTA via a centered difference approximation to equation (1):

$$\mathbf{N}(t) \approx \frac{\mathbf{x}(t+\delta) - \mathbf{x}(t-\delta)}{2\delta} - \mathbf{L}\mathbf{x}(t) \quad (3)$$

with $\delta = 1$ wk. The centered difference emulates calculus appropriate for continuous systems with noise that is only approximately white [*Gardiner*, 1984; *Kloeden and Platen*, 1992] (see also supplementary material). For our rapidly varying SSTs, we rely on the Optimal Interpolated SST data set (OISST [*Reynolds et al.*, 2002]) because there is no sub-monthly version of ERSST. Our comparison of monthly OISST and ERSST (see supplemental material) showed the two time series were statistically indistinguishable during the 22-year period of overlap.

We continued investigating the stochastic forcing of coarse-grained SST_{NTA} from ERSST using weekly OISST as follows. Weekly OISST anomalies in the period 1990-2012 (1186 maps) were consolidated onto a 4° x 10° latitude by longitude grid, projected onto the leading 20 ERSST EOFs, and detrended by removing the ERSST trends from the leading two PCs. Then, we estimated $\mathbf{N}(t)$ as described in equation (3). We denote the resulting stochastic forcing in the NTA region as $N_{\mathrm{NTA}}(t)$.

3. Quantifying that "unpredictable" SST component

To quantify the unpredictable component of tropical SST, we integrate equation (1), yielding

$$\mathbf{x}(t+\tau) = \exp(\mathbf{L}\tau)\mathbf{x}(t) + \int_0^{\tau} \exp\left(\mathbf{L}[\tau - s]\right)\mathbf{N}(t+s)\,ds \,. \tag{4}$$

The first term on the right hand side is the LIM forecast (or, in our case, the "hindcast") of $\mathbf{x}(t+\tau)$ based on the slow dynamics encapsulated in $\mathbf{L}$. The second term is the unpredictable (on seasonal timescales) component of SST generated between time t and $t + \tau$ and comprises not only the effect of local stochastic forcing, but also the redistribution of stochastically induced variability by deterministic dynamics.

4. NAO's contribution to stochastic forcing

The NAO index we used was generated at NOAA/ESRL/Physical Science Division by spatially smoothing NCAR/NCEP reanalyses of 500 hPa geopotential heights to emphasize large scales, averaging them in two domains (55°N - 70°N, 70°W - 10°W and 35°N - 45°N, 70°W - 10°W), computing daily anomalies from the 1981-2010 climatology, and subtracting northern values from southern values. This generated a time series that, being based on upper-air analyses, includes NAO effects related to the large-scale circulation.

Figure 2 shows 500 hPa geopotential height patterns composited on values of the monthly stochastic forcing $N_{\mathrm{NTA}}(t)$ tailward of one standard deviation. The main Atlantic feature is a high/low couplet roughly coincident with the domains used to generate the NAO index, indicating that NAO-like variability is strongly associated with stochastic forcing in the NTA region. Figure 2 suggests that the NAO in this context is part of a larger wave train extending into northern Scandinavia. It also suggests that the negative phase of NAO dominates the pattern somewhat more during positive extremes of $N_{\mathrm{NTA}}(t)$ than does the positive phase of NAO during negative extremes of $N_{\mathrm{NTA}}(t)$.

It is not just $N_{\mathrm{NTA}}(t)$ that provides forcing in equation (4), but rather the entire vector $\mathbf{N}(t)$. The NAO contribution to $\mathbf{N}$(t) is represented by $\mathbf{R}\eta(t)$, where R is estimated as follows. We averaged the daily NAO to generate a weekly time series $\eta(t)$ contemporaneous with OISST. Stratifying data by month and regressing each month's $\mathbf{N}(t)$ (equation 3) onto $\eta(t)$ yielded annually periodic regression vectors $\mathbf{R}_{\mathbf{month}}$, which relate the NAO to the pattern of SST forcing. Three passes of a 3-month boxcar filter revealed that each component of $\mathbf{R}_{\mathbf{month}}$ contained a systematic annual cycle; weekly values $\mathbf{R}_{\mathbf{week}}$ were therefore estimated by linear interpolation.

Figure 3 illustrates the annual cycle of $\mathbf{R}_{\mathbf{week}}$. The largest sensitivity to the NAO index is in the north Atlantic, especially during boreal summer (Fig. 3c). Even though strongest sensitivity is north of our NTA region, which is outlined in red, the linear deterministic dynamics redistributes the forcing over time (cf. the integral in equation (4)). In the north tropical and subtropical Atlantic, surface winds composited on NAO extremes (not shown) indicate that trade winds weaken during the negative phase of the NAO and strengthen during the positive phase, directly affecting SST_{NTA}. We defer to future work the apparent presence of NAO forcing in other ocean basins. The NAO contribution to the unpredictable component of NTA SST can be estimated by using $\mathbf{R}_{\mathbf{week}}\eta(t)$ in equation (4) instead of $\mathbf{N}(t)$, with the integral approximated by trapezoidal rule.

5. Evidence of the NAO's effects on SSTs in the North Tropical Atlantic

We earlier noted that the autocorrelation functions of weekly $SSTA_{NTA}$ and NAO have quite different time scales. The correlations between weekly $SSTA_{NTA}$ and LIM hindcasts thereof (i.e. based on equation (3) using only the first term on the right-hand side) have more skill than a univariate AR1 process such as $SSTA_{NTA}$ from 12 through 48 weeks (Fig. 1a). Because the data involved are not necessarily independent, this curve can be considered an upper bound of hindcast skill. We can include the NAO's contribution to stochastic forcing by adding the integral term in equation (3), with $\mathbf{N}(t)$ replaced with $\mathbf{R}_{\mathbf{week}}\eta(t)$, and use the correlations between weekly $SSTA_{NTA}$ and LIM hindcasts in Fig. 1a as a benchmark to show the fraction of the unpredictable component of SST_{NTA} due to rapidly varying NAO. The correlations between weekly $SSTA_{NTA}$ and the LIM hindcasts plus the NAO contribution (Fig. 1b) show that much of the unpredictable component can be attributed to the NAO; at a lead of 48 weeks, the correlation is increased from the baseline 0.37 to 0.56.

The NAO is most active during the boreal winter (DJF) and spring (MAM). To identify how much of the contribution to the integral in equation (3) is attributable to NAO during these seasons, Fig. 1b also shows the correlations between $SSTA_{NTA}$ and LIM hindcasts augmented by two other versions of the integral term: one using contributions from every season except DJF, and one using contributions from only boreal summer (JJA) and fall (SON). The greatest augmentation to the correlation between $SSTA_{NTA}$ and LIM hindcasts is from forcing during DJF and MAM. However, as shown in Fig. 4, unpredictable SST variability generated during these seasons is persisted by the deterministic dynamics and affects the predictability of $SSTA_{NTA}$ during every season of the year, particularly at long lead times. Clearly, the disruption of JJA and SON predictability is more strongly affected by the NAO in MAM than in DJF.

5. Conclusions and Discussion

While the NAO does not account for all of the stochastic forcing of $SSTA_{NTA}$, it does account for an impressive amount and, although the NAO is strongest during the boreal winter and spring, its convolution with the deterministic dynamics maintains its importance throughout the year. The decorrelation time of the NAO is on the order of one or two weeks, but the NAO's cumulative effect when convolved with the longer-scale deterministic dynamics of tropical SSTs is strongly associated with inter-seasonal evolution of tropical SST. Thus, long timescale predictions of $SSTA_{NTA}$ require accurate simulation of fast dynamics. The NAO's importance to $SSTA_{NTA}$ evolution cannot be discerned through traditional correlation analysis since deterministic dynamics redistributes NAO-induced variability during the forecast period.

Our results also show how *forecast skill* of SST anomalies obeying a stochastically forced linear system can be sensitive to the annual cycle even though *forecasts* themselves may not be. Passage of the "tau-test" (see *Penland and Sardeshmukh* [1995] and supplement for details) suggests weak, if any, dependence of **L** (the slow dynamics) on the annual cycle. However, since forecast skill is strongly linked to stochastic forcing, skill can be seasonally dependent if the variance of stochastic forcing depends on the annual cycle, as is the case with NAO.

Our study provides an estimate of how a perfect forecast of the NAO might improve SST forecasts. Statistical forecasts rely upon correlation, and the short autocorrelation of the NAO precludes useful statistical forecasts of that quantity at seasonal leads. However, the relation between NAO and $SSTA_{NTA}$ might be exploited by GCMs to the extent allowed by dynamical chaos. If fast nonlinear dynamics resolved by GCMs could be predicted accurately enough that some of what seasonally averaged SSTs "see" as stochastic forcing could be deterministically predicted, the lead times at which forecasts of $SSTA_{NTA}$ are skillful might exceed those indicated in Fig. 1a.

This study suggests extensive follow-up work. Both El Niño and the NAO have significant effect on $SSTA_{NTA}$; are they related? What are the roles of other teleconnections, and how much of that dynamical interaction is stochastic on seasonal timescales? For example, much of the effect of ENSO on $SSTA_{NTA}$ is represented in the deterministic operator **L** [*Penland and Matrosova,* 1998, 2006], but ENSO is also dynamically related to the Pacific- North American pattern (PNA), which itself affects $SSTA_{NTA}$. We found insignificant correlations between PNA and NAO at daily, weekly, and monthly timescales (not shown), so we believe that any joint effect of these indices would also be represented in **L**, but this remains to be proved. Can physical mechanisms responsible for the stochastic forcing be identified through further data analysis, as in *Frankignoul et al.* [1998]? We expect the multi-scale statistical analysis introduced here, particularly when combined with other dynamically based statistical methods, to provide a valuable tool for investigating these and other unresolved issues.

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References

Benedict, J.L., S. Lee, and S.B. Feldstein (2004), Synoptic view of the North Atlantic Oscillation, *J. Atmos. Sci., 61*(2), 121-144.

Czaja, A., P. van der Vaart, and J. Marshall (2002), A diagnostic study of the role of remote forcing in tropical Atlantic variability, *J. Climate*, *15*(22), 3280-3290.

Elsner, J.B., S.E. Strazzo, T.H. Jagger, T. LaRow, and M. Zhao (2013), Sensitivity of limiting hurricane intensity to SST in the Atlantic from observations and GCMs. *J. Climate, 26*(16), 5949-5957.

Enfield, D.B. (1996), Relationships of inter-American rainfall to tropical Pacific and Atlantic variability. *Geophys. Res. Lett.*, *23*(23), 3305-3308.

Enfield, D. B., and D. A. Mayer (1997), Tropical Atlantic sea surface temperature variability and its relation to El Niño-Southern Oscillation, *J. Geophys. Res.*, *102*(C1), 929-945.

Feldstein, S. B. (2000), The timescale, power spectra, and climate noise properties of teleconnection patterns. *J. Climate*, *13*(24), 4430–4440.

Feldstein, S.B. (2007), The dynamics of the North Atlantic Oscillation during the summer season. *Quart. J. Roy. Meteorol. Soc., 133*(627), 1509-1518.

Frankignoul, C., and K. Hasselmann (1977), Stochastic climate models, Part II Application to sea-surface temperature anomalies and thermocline variability, *Tellus*, *29*(4), 289-305.

Frankignoul, C., A. Czaja, and B. L'Heveder (1998), Air-sea feedback in the North Atlantic and surface boundary conditions for ocean models. *J. Climate*, *11*(9), 2310-2324.

Franzke, C., S. Lee, and S. B. Feldstein (2004), Is the North Atlantic Oscillation a breaking wave? *J. Atmos. Sci.*, *61*(2), 145-160.

Fuenzalida, H., and B. Rosenblüth (1986), Distortion effects of the anomaly method of removing seasonal or diurnal variations from climatological time series, *J. Climate Appl. Meteorol.*, *25*(6), 728-731. doi 10.1175/1520-0450(1986)025<0728:DEOTAM>2.0.CO;2

García, A.L., M.M Mansour, G.C. Lie,and E. Clementi (1987), Numerical integration of the fluctuating hydrodynamic equations, *J. Stat. Phys.*, *47*(1/2), 209-228.

Gardiner, C.W. (1984), *Handbook of Stochastic Methods*. Springer-Verlag, Berlin.

Goldenberg, S.B., and L.J. Shapiro (1996), Physical mechanisms for the association of El Niño and West African rainfall with Atlantic major Hurricane activity, *J. Climate*, *9*(6), 1169-1187.

Hasselmann, K. (1976), Stochastic climate models Part I. Theory, *Tellus*, *28*(6), 473-485.

Hastenrath, S. (1984), Interannual variability and annual cycle: Mechanisms of circulation and climate in the tropical Atlantic sector. *Mon. Wea. Rev.*, *112*(6), 1097-1117.

Khasminskii, R. Z. (1966), A limit theorem for solutions of differential equations with random right-hand sides. *Theory Prob. Applications*, *11*, 390-406.

Kloeden, P., and E. Platen (1992), *Numerical Solution of Stochastic Differential Equations*. Springer Verlag, Berlin, 632pp.

Kushnir, Y., W.A. Robinson, P. Chang, A.W. Robertson (2006) The physical basis for predicting Atlantic sector seasonal-to-interannual climate variability. *J. Climate*, *19*(23), 5943-5970.

Kushnir, Y., R. Seager, M. Ting, N. Naik, and J. Nakamura (2010), Mechanisms of tropical Atlantic SST influence on North American precipitation variability. *J. Climate*, *23*(21), 5610-5628.

Lee, S.-K., D.B. Enfield, and C. Wang (2008), Why do some El Niños have no impact on tropical North Atlantic SST? *Geophys. Res. Lett.*, *35*, L16705, doi:10.1029/2008GL034734.

Nobre, P., and J. Shukla (1996) Variations of sea surface temperature, wind stress and rainfall over the tropical Atlantic and South America. *J. Climate, 9*(10), 2464–2479.

Papanicolaou, G.C., and W. Kohler (1974), Asymptotic theory of mixing stochastic ordinary differential equations. *Commun. Pure Appl. Math.*, *27*, 641-668.

Penland, C., and L. Matrosova (1998), Prediction of tropical Atlantic sea surface temperatures using linear inverse modeling, *J. Climate*, *11*(3), 483-496.

Penland, C., and L. Matrosova (2006), Studies of El Nino and interdecadal variability in tropical sea surface temperatures using a nonnormal filter. *J. Climate*, **19**, 5796-5815.

Penland, C., and P. D. Sardeshmukh (1995), The optimal growth of tropical sea-surface temperature anomalies, *J. Climate*, *8*(8), 1999-2024.

Polo, I., B. Rodríguez-Fonseca, T. Losada, J. García-Serrano, (2008), Tropical Atlantic variability modes (1979–2002). Part I: Time-evolving SST modes related to West African rainfall. *J. Climate, 21*(24), 6457-6475.

Reynolds, R. W., N. A. Rayner, T. M. Smith, D. C. Stokes, and W. Wang (2002), An improved in situ and satellite SST analysis for climate, *J. Climate*, *15*, 1609-1625.

Saha, S., and CoAuthors (2006), The NCEP climate forecast system, *J. Climate*, *19*(15), 3483-3517.

Saravanan, R., and P. Chang (2000), Interaction between tropical Atlantic variability and El Niño–Southern Oscillation, *J. Climate*, *13*(13), 2177-2194.

Smirnov, D., and D.J. Vimont (2013), Extratropical forcing of tropical Atlantic variability during boreal summer and fall, *J. Climate*, *25*(6), 2056-2076.

Smith, T.M., R.W. Reynolds, T.C. Peterson, J. Lawrimore (2008), Improvements to NOAA's historical merged land–ocean surface temperature analysis (1880–2006), *J. Climate, 21*(10), 2283-2296.

Stockdale, T.N., M. A. Balmaseda, and A. Vidard (2006), Tropical Atlantic SST prediction with coupled ocean-atmosphere GCMs. *J. Climate*, *19*(23), 6047-6061.

Wu, L., and A. Liu (2002), Is tropical Atlantic variability driven by the North Atlantic Oscillation? *Geophys. Res. Lett.*, *29*(13), 1653, DOI: 10.1029/2002GL014939

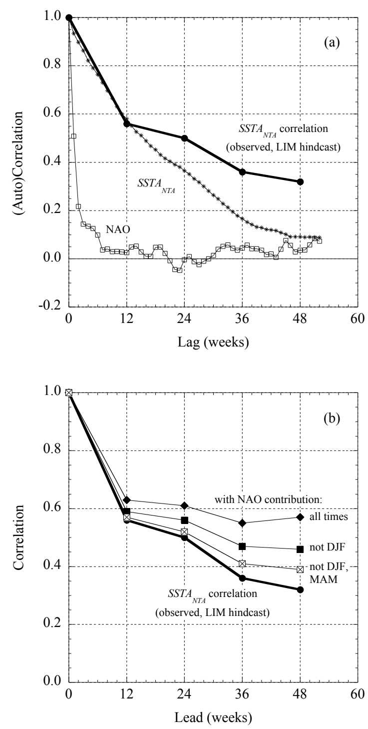

Figure 1. (a) Autocorrelation of the weekly NAO (squares) and $SSTA_{NTA}$ (stars). Also shown (thick line) is the correlation between observations and LIM hindcasts of weekly $SSTA_{NTA}$ at 12, 24, 36, and 48 weeks. (b) The thick line repeats the correlation between observations and LIM hindcasts of weekly $SSTA_{NTA}$. The other curves show the correlation between observed weekly $SSTA_{NTA}$ and LIM hindcasts thereof modified to include contribution to $SSTA_{NTA}$ from the NAO (cf. equation (3)). Filled diamonds: including NAO during the entire year. Filled squares: neglecting NAO contributions from boreal winter. Open squares with crosses: neglecting NAO contributions from boreal winter and spring.

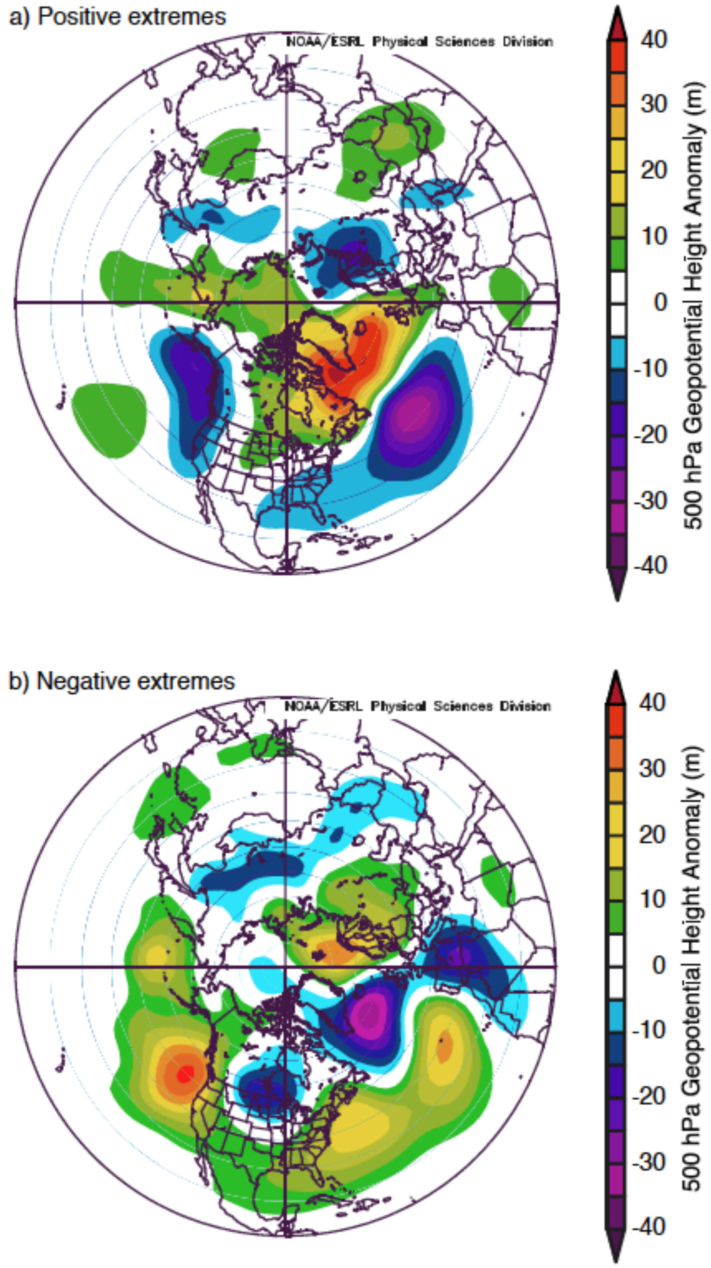

Figure 2. Composite daily 500mb heights corresponding to (a) positive (N=196) and (b) negative (N=185) extremes of empirically estimated stochastic forcing of $SSTA_{NTA}$. Extremes are defined as excursions beyond plus/minus one standard deviation.

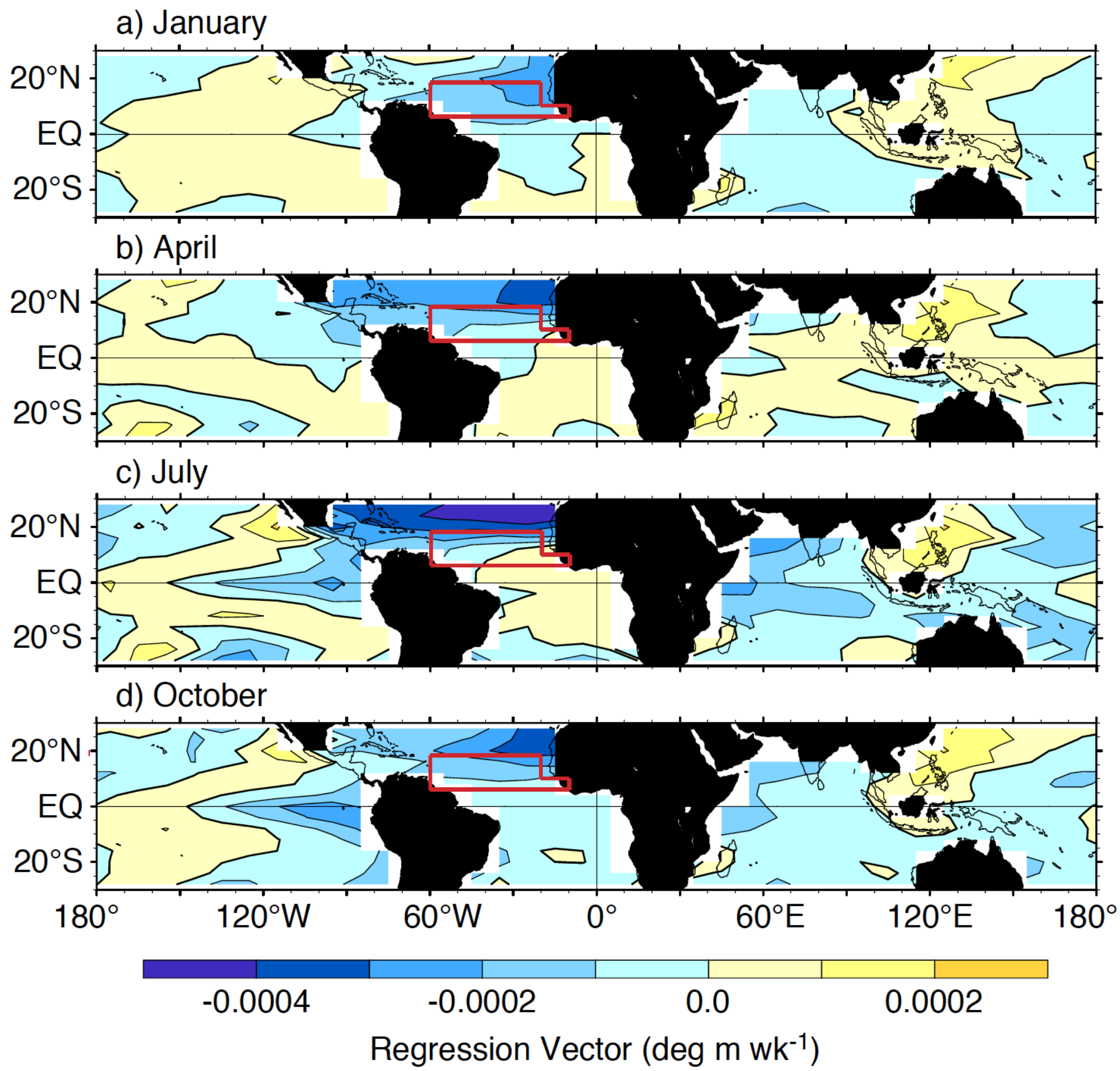

Figure 3. Seasonal variation of the regression vector **R**. (a) January, (b) April, (c) July, (d) October. See text for details. The NTA region is outlined in red.

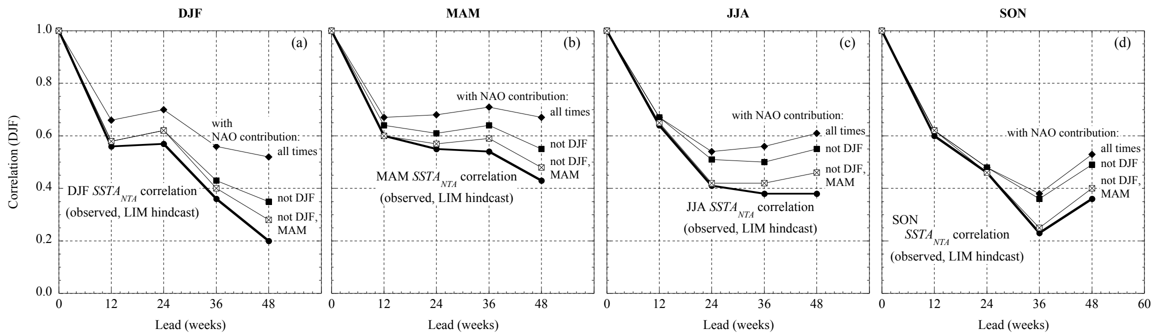

Figure 4. As in Figure 1b, but stratified by verification season. (a) DJF, (b) MAM, (c) JJA, (d) SON.

Supplemental Material

1. Constructing the linear operator

We used the coarse-grained SST anomalies $\mathbf{X}$ to estimate the linear operator $\mathbf{L}$. Defining $\mathbf{C}(\tau)$ as the autocorrelation matrix of $\mathbf{X}$ at lag τ, then

$$\mathbf{C}(\tau) = \exp(\mathbf{L}\tau)\, \mathbf{C}(0). \qquad \text{(S1)}$$

Hence, by choosing a lag τ_o and estimating $\mathbf{C}(\tau_o)$ and $\mathbf{C}(0)$ from data, we estimate $\mathbf{L}$ from the eigenstructure of $\mathbf{C}(\tau)\mathbf{C}^{-1}(0)$ [*Penland and Sardeshmukh*, 1995; PS95 hereafter]. Estimates of $\mathbf{L}$ that are independent of τ_o imply linear dynamics for $\mathbf{X}$, though Nyquist modes, short data records and observational errors may prevent passage of this "tau-test" even when equation (2) is valid [PS95]. The tau-test also fails if $\mathbf{L}$ is nonstationary, e.g., if it depends on the annual cycle. LIM forecasts of $\mathbf{X}$ [PS95] have shown that linear models of tropical SST obey the tau test reasonably well.

LIM forecasts of tropical SSTA at lead τ are obtained by applying a "Green function" matrix $\mathbf{G}(\tau)$ to an initial condition $\mathbf{X}(t_0)$, where

$$\mathbf{G}(\tau) = \exp(\mathbf{L}\tau). \qquad \text{(S2)}$$

Simple LIM forecasts of seasonal SST in the tropical strip are competitive with those made by operational forecasting systems [*Saha et al.*, 2006], and skillful LIM forecasts of $SSTA_{NTA}$ have been available for over a decade [*Penland and Matrosova*, 1998]. Three-month running mean SSTs have been used to estimate $\mathbf{L}$ for real-time forecasts (rather than straight seasonal coarse-graining) to minimize computational round-off errors. Another issue is one of smoothness: LIM attempts to identify multivariate tendencies in the coarse-grained data, and a running mean facilitates this.

The matrix **L** was estimated to conform with forecast guidelines furnished by the National Oceanic and Atmospheric Administration (NOAA). The 1981 - 2010 climatology was subtracted from NOAA Extended Reconstructed SST (ERSST) V3b monthly data [*Smith et al.*, 2008], 1950 to 2000, to form anomalies [*Fuenzalida and Rosenblüth*, 1986]. Anomalies in the tropical strip between 30°N and 30°S were consolidated onto a 4^{o} x 10° latitude by longitude grid, subjected to a three-month running mean, and projected onto the leading 20 Empirical Orthogonal Functions (EOFs), explaining 89% of the variance. The two leading EOFs, together explaining 58% of the variance, each bear some resemblance to a mature El Niño pattern in the eastern Pacific, with the second EOF more closely confined to the equator. The principal component (PC) time series of these two EOFs exhibit highly significant trends, though in opposite senses; the linear trend in each of these PCs was removed. The other PCs do not show strongly significant trends. Using these 20 time series as a basis, the linear operator **L** was estimated using equation (S1).

Resulting SST forecasts, including those in the NTA region (c.f. red box in Fig. C), are found at http://www.esrl.noaa.gov/psd/forecasts/sstlim/Seas.html. If the prediction model were perfect, stochastic forcing, which does not enter into the forecast itself, would dominate the forecast error (equation (3)), with no difference in forecast error statistics made with dependent and independent data.

2. Comparing ERSST and OISST

OISST comes in both monthly and weekly versions, but only from 1981 to present. This duration is too short to give accurate estimations of the coarse-grained dynamics represented by

L, but OISST is an adequate approximation of ERSST for estimating the rapidly varying dynamics **N**(t). To show this, we prepared anomalies of monthly OISST as we did with ERSST, projecting them onto ERSST EOFs and removing the same linear trends. The resulting time series were statistically indistinguishable from the ERSST time series during the period of overlap, i.e., 1990-2012.

3. Estimating forcing

Although the statistics of the stochastic forcing is a product of LIM, the time series of stochastic forcing is not. An estimation of such a time series is the residual of subtracting **Lx** from a finite difference approximation to the derivative in equation (1). This seems obvious, but there is a mathematical subtlety required by the physics: it is required to recall that white noise is only an approximation and that the system we consider here is continuous, rather than discrete. Recognizing that nature imposes on continuous, possibly discretized, stochastic physical systems (i.e., "Stratonovich" systems) mathematical rules that are different for systems that are intrinsically discrete [e.g., *Gardiner*, 1984], we must use a finite difference that yields a nonzero contemporaneous correlation between **X** and the additive white noise forcing $\boldsymbol{\xi}$ in equation (2). In fact, $\langle \mathbf{X}\boldsymbol{\xi}^T \rangle = \mathbf{Q}/2$, where **Q** is the covariance matrix of the noise: $\langle \boldsymbol{\xi}\,\boldsymbol{\xi}^T \rangle dt = \mathbf{Q}$ [*García et al.*, 1987]. It is easily shown using the fluctuation-dissipation theorem of the second kind that a central difference approximation to this derivative provides this relation in the limit that the time step goes to zero. Since a monthly time step, particularly representing data subjected to a three-month running mean, cannot remotely satisfy the approximation of small time step, we employ weekly data to estimate the stochastic forcing as in equation (3).