Appendix to 'Probabilistic Precipitation Forecast Postprocessing Using Quantile Mapping and Rank-weighted Best-Member Dressing: Description of the CSGD method.

Here we describe the modifications of the Censored, Shifted Gamma Distribution method by Scheuerer and Hamill (2015) made in order to address the particular challenges of this study.

The original CSGD approach started with quantile mapping the forecasts, enlarging the ensemble by including forecasts at nearby grid points, and calculating a number of ensemble statistics. We perform the same three steps here, but in doing so we proceed as described in Section 3b of this paper and *not* as proposed by SH15. Some of the ensemble statistics considered here are also slightly different: we still use the ensemble mean $\bar{\tilde{x}}^f$, and the fraction of non-zero ensemble members $F\hat{N}Z$, but as a measure of ensemble spread we use the standard deviation $\sigma(\tilde{x}^f)$ instead of the mean absolute difference. We do not use precipitable water as a predictor here.

The second step in the procedure described by SH15 is to fit climatological CSGD parameters - separately for each month, each lead time, and each grid point - to the analyzed precipitation amounts used for training. Here, we do this using analysis data from 2002 to 2017 and a slightly different model fitting approach. As in SH15, for each month the training data is composed of the 45 days before and after the 15th of each month. From these data, we calculate the climatological mean $\bar{y}_{cl}$ and the climatological fraction of zero (here < 0.254 mm) precipitation analyses $\hat{FZ}_{cl}$. We choose the shape parameter k_{cl} of the climatological CSGD such that the continuous ranked probability score (CRPS) over the training sample is minimized, while the scale parameter θ_{cl} and the shift parameter δ_{cl} are chosen such that the CSGD defined by those three parameters has the prescribed climatological mean $\bar{y}_{cl}$ and climatological fraction zero $\hat{FZ}_{cl}$. Specifically, denote by F_k the cumulative distribution function (CDF) and by f_k the probability density function of a gamma distribution function with shape parameter k. Let F_k^{-1} be the inverse CDF and define $q_0 := F_k^{-1}(\hat{FZ}_{cl})$. For a given k, the parameters θ_{cl} and δ_{cl} can then be calculated via

$$\theta_{cl} = \frac{\bar{y}_{cl}}{k\big(1 - \hat{FZ}_{cl} + f_{k+1}(q_0)\big) - q_0\big(1 - \hat{FZ}_{cl}\big)}, \quad \delta_{cl} = -\theta_{cl}\, q_0$$

The climatological shape parameter k_{cl} is then found via CRPS minimization as described in SH15. In contrast to their original suggestion, minimization is now performed over a 1-dimensional (instead of 3-dimensional) parameter space, and is therefore computationally more efficient. Moreover, the three quantities k_{cl}, $\bar{y}_{cl}$, and $\hat{FZ}_{cl}$ have an intuitive interpretation and can be assumed to have a smooth annual cycle. We can therefore linearly interpolate them from the 15th of each month to every single day of the year, and calculate the climatological CSGD parameters μ_{cl}, σ_{cl} and δ_{cl} from eqs. (1), (2) in SH15 from the interpolated values of k_{cl}, $\bar{y}_{cl}$, and $\hat{FZ}_{cl}$.

The final step in SH15 is to link the CSGD parameters of the calibrated forecast distribution at grid point *s* to the ensemble statistics at *s* defined in the first step. In the present setup, we use the following regression equations

$$\mu_s = \frac{\mu_{cl,s}}{\alpha_1} \operatorname{log1p}\left[\operatorname{expm1}(\alpha_1)\left(\exp(-\rho_s/\alpha_2) + \alpha_3 F\hat{N}Z_s + \alpha_4 \rho_s \frac{\bar{\tilde{x}}_s^f}{\bar{\tilde{x}}_{cl,s}^f}\right)\right],$$

$$\sigma_s = \alpha_5\, \sigma_{cl,s} \sqrt{1-\rho_s^2}\, \sqrt{\frac{\mu_s}{\mu_{cl,s}}} + \alpha_6\, \sigma\left(\tilde{x}_s^f\right),$$

$$\delta_s = \delta_{cl,s},$$

where $\operatorname{log1p}(x) = \log(1+x)$, $\operatorname{expm1}(x) = exp(x) - 1$, and $\bar{\tilde{x}}_{cl,s}^f$ is the climatological average of the ensemble mean $\bar{\tilde{x}}_s^f$ of the quantile-mapped forecasts. Due to the quantile mapping, we can assume $\bar{\tilde{x}}_{cl,s}^f = \bar{y}_{cl,s}$, where $\bar{y}_{cl,s}$ is the climatological mean of the analyzed precipitation at s. The regression equations above differ from those used in SH15 in a number of ways. First, there are only six regression parameters since there is no precipitable water predictor and since the heteroscedasticity parameter was fixed to 0.5 (a simplification suggested by SH15). Second, due to the limited training sample size and the challenges that come with the estimation of a rather complex model, the regression parameters $\alpha_1, \ldots, \alpha_6$ are assumed constant across the entire domain, and estimated in a single CRPS minimization. As explained in SH15, one of the motivations for incorporating the climatological CSGD parameters $\mu_{cl,s}$ and $\sigma_{cl,s}$ into the regression equations is that this removes some of the local characteristics. One characteristic that is not captured that way is the local forecast skill of the NWP model, which can vary substantially across the domain. By assuming $\alpha_1, ..., \alpha_6$ constant in space, these parameters cannot account for spatially varying skill either, and this motivated the inclusion of a spatially varying skill parameter ρ_s. This parameter is defined as the correlation between the mean of square-root transformed, quantile-mapped ensemble forecasts and square-root transformed analyzed precipitation amounts. The calculation is performed with the same training sample used for estimating the regression parameters, but each analysis grid point was supplemented by the best 19 supplemental locations found as described in Section 3a. The particular way of including ρ_s in the regression equations above is motivated by standard regression theory. If two variables have correlation ρ, the slope parameter for regressing one of them on the other is proportional to ρ, and the unexplained variance is proportional $1 - \rho^2$. Relating the intercept parameter to ρ_s is more challenging in the present context; we chose the expression $\exp(-\rho_s/\alpha_2)$ since it ensures that the intercept is always positive, is equal to 1 if there is zero skill, and tends to zero as skill increases. With these three changes, we hope to strike a balance between parsimonious parameterization - which is necessary in a situation with limited training data - and sufficient flexibility to address local characteristics of the different analysis grid points within the CONUS.